Math 10B - Calculus of Several Variables II
Quiz 2
April 15, 2011

Name: _____

Total
/10

Evaluate the integral:

$$\iiint_{[1,e]\times[1,e]\times[1,e]} \frac{1}{xyz}\,dV.$$

Solution. The order of integration actually does not matter here (since the region is a cube), so we will proceed as follows:

$$
\begin{aligned}
\iiint_{[1,e]\times[1,e]\times[1,e]} \frac{1}{xyz}\,dV
&= \int_1^e \int_1^e \int_1^e \frac{1}{xyz}\,dx\,dy\,dz \\
&= \int_1^e \int_1^e \left(\frac{\ln|x|}{yz}\right)\Big|_1^e \,dy\,dz \\
&= \int_1^e \int_1^e \left(\frac{1}{yz}(\ln|e| - \ln|1|)\right)\,dy\,dz \\
&= \int_1^e \int_1^e \left(\frac{1}{yz}(1 - 0)\right)\,dy\,dz \\
&= \int_1^e \int_1^e \left(\frac{1}{yz}\right)\,dy\,dz \\
&= \int_1^e \left(\frac{1}{z}\ln|y|\right)\Big|_1^e \,dz \\
&= \int_1^e \left(\frac{1}{z}(\ln|e| - \ln|1|)\right)\,dz \\
&= \int_1^e \frac{1}{z}\,dz \\
&= (\ln|z|)\,|_1^e \\
&= \ln|e| - \ln|1| = 1 - 0 = 1
\end{aligned}
$$

□